TACTILE ALGEBRA TILES

UEB COMPLIANT

APH

Catalog No. 1-08410-00

WARNING:
CHOKING HAZARD - Small parts.
Not intended for children ages 5 and under without adult supervision.

Tactile Algebra Tiles

Algebra tiles is a mathematical manipulative that provides students with concrete models for approaching abstract algebraic concepts and procedures. The tool makes it possible for students to visualize expressions and equations and thus helps meet their diverse needs in algebra study.

For students who struggle with the abstract nature of algebra, algebra tiles can help them access the algebra related content laid out in the Common Core State Standards for Mathematics (CCSSM). The tiles can be used by students in elementary through high school for adding, subtracting, and multiplying integers, simplifying expressions, solving linear and quadratic equations, and for adding, subtracting, multiplying, and factoring polynomials. These subjects relate to the standards of many CCSSM domains such as Operations and Algebraic Thinking, Number and Operations, The Number System, and Expressions and Equations. Please refer to the CCSSM for more details.

Tactile Algebra Tiles is an accessible version of these algebra tiles, specifically designed for students with blindness and low vision. It includes magnetic tiles that the students can manipulate on a steel board, and uses tactile symbols to help students differentiate between tiles. Colors are chosen with consideration for the needs of students who have low vision.

Tactile Algebra Tiles is consistent with regular algebra tiles used by sighted students, and this allows students who are visually impaired to work together with their classmates in inclusive classrooms. We hope this product can help make your teaching and learning experience an enjoyable one.

Product Components

The Tactile Algebra Tiles kit consists of the following components:

- Steel board - two foldable boards on which students can place magnetic tiles. Students can use one board as a working board and the other for storage.
- Small lime square tile (32 pieces) - small square tile with a tactile square symbol at the center. These represent the numerical value "+1."
- Small red square tile (32 pieces) - small square tile with a raised dot at the center. These represent the numerical value "−1."
- Violet rectangle tile (12 pieces) - rectangle tile with a tactile rectangular symbol at the center.

These represent the numerical value "+x."

- Red rectangle tile (12 pieces) - rectangle tile with a row of raised dots at the center. These represent the numerical value "−x."
- Large blue square tile (6 pieces) - large square tile with a tactile square symbol at the center. These represent the numerical value "$+x^2$."
- Large red square tile (6 pieces) - large square tile with a dotted square symbol at the center. These represent the numerical value "$-x^2$."
- Black separation bar (2 pieces) - two long, narrow pieces that can be used to separate two sides of an equation.

Examples of Use

The following section provides several examples of how to use Tactile Algebra Tiles to solve algebraic problems; however, it is not a comprehensive guide on the use of algebra tiles. For further details about how to use algebra tiles, please refer to other resources.

The concept of "zero pair" is often used in algebra tile illustrations. Because the sum of a number and its opposite is zero, a positive tile and a negative tile paired together represent zero and are called a zero pair. For example, one small lime square and one small red square form a zero pair, while three violet rectangles and three red rectangles make up three zero pairs. When using algebra tiles to solve algebraic problems, zero pairs can be added to or removed from a mathematical expression without changing the value of the expression.

In all examples, lime, violet, and blue tiles are used to represent positive values and red tiles are used to represent negative values.

1. Adding and subtracting integers

To illustrate the concept of adding two positive numbers, such as 3 + 6, have the student show two groups of small lime squares, one with three tiles representing positive 3, and the other with six tiles representing positive 6 (Picture 1). The student will then add the two groups by moving the tiles together. Have the student count the tiles to get the answer, 3 + 6 = 9.

Picture 1: Modeling two positive numbers 3 and 6 using Tactile Algebra Tiles

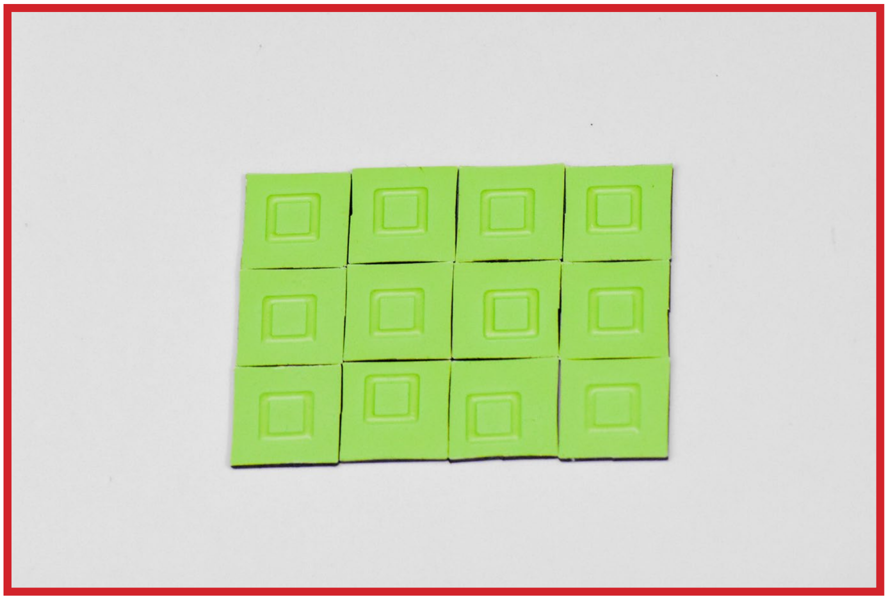
To illustrate the concept of adding a positive number and a negative number, such as 3 + (−7), have the student show two groups of small squares, one with three lime squares representing positive 3, and the other with seven red squares representing negative 7. To add the two groups, move them together. Because three lime squares and three red squares make up three zero pairs, they can be removed without changing the total value of the expression. Since there are four small red squares remaining, the answer to the problem is −4.

To illustrate subtraction problems such as −5 − (−2), first have the student use one group of five small red squares to represent −5. To subtract −2, remove two small red squares. Three red squares are left, so the answer is −3.

To illustrate subtraction problems such as −2 − 5, remind the student that to subtract a number, you can add its opposite. Therefore, the problem becomes −2 + (−5). Then, have the student show two groups of small red squares, one group representing −2 and the other representing −5. The student will move the tiles together and get the answer, −7. Another way to illustrate this problem is to use zero pairs. First, have the student show two small red squares which represent −2. Next, add five zero pairs of small red and lime squares so that there are enough lime squares to subtract. Notice that adding zero pairs does not change the total value. Then, remove five lime squares to subtract 5. The answer is still −7.

2. Multiplication of integers

To illustrate the multiplication problem 3 × 4, place three groups of four small lime squares together to form a rectangle. The vertical and horizontal sides of the rectangle represent the two factors 3 and 4 (Picture 2). Count the number of tiles in the rectangle to get its area, 12, which is the answer to the problem.



Picture 2: Illustration of 3 × 4 using Tactile Algebra Tiles

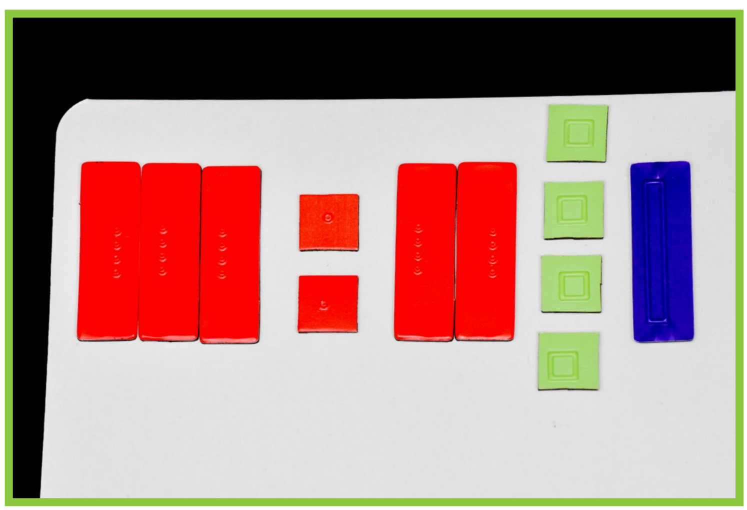
To illustrate the multiplication problem $3 \times (-4)$, use three groups of four small red squares to form a three-by-four rectangle. Count the total number of tiles in the rectangle. This time, the answer is −12. Multiplication problems such as $(-3) \times 4$ can be treated as $4 \times (-3)$ by reminding the student that the order in which we multiply does not matter.

To illustrate the multiplication problem $(-3) \times (-4)$, remind the student that the "–" sign can mean "opposite of." Then, the problem can be treated as the opposite of $3 \times (-4)$. Thus, the answer is 12.

3. Simplifying algebraic expressions

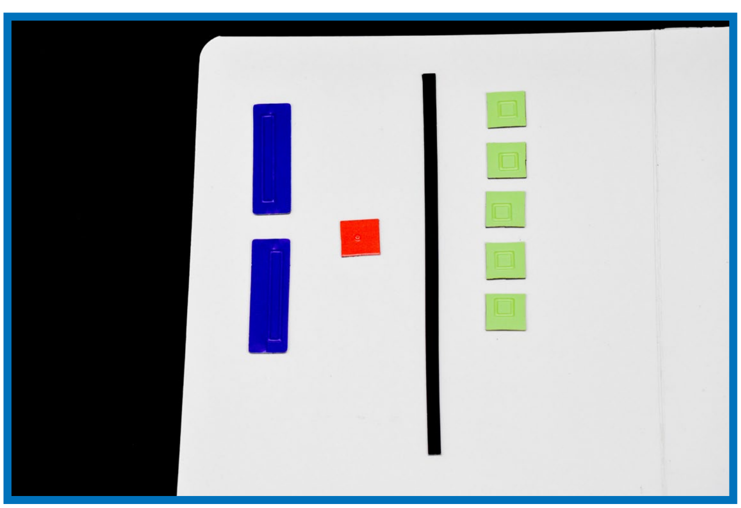
Model the expression $-3x - 2 - 2x + 4 + x$ using algebra tiles. Because subtracting is the same as adding the opposite, the expression can be treated as $-3x + (-2) + (-2x) + 4 + x$. Thus, have the student place three red rectangles, two small red squares, two red rectangles, four small lime squares, and one violet rectangle together (Picture 3). Then, have the student move like terms together. That is, move three red rectangles, two red rectangles, and one violet rectangle together and two small red squares and four small lime squares together. Because the violet rectangle and one of the red rectangles form a zero pair, they can be removed without changing the total value of the expression. Similarly, two small red squares when grouped with two lime squares form two zero pairs and can be removed too. Because there are only four red rectangles and two small lime squares left, the simplified expression is $-4x + 2$.

Picture 3: Modeling expression $-3x + (-2) + (-2x) + 4 + x$ using Tactile Algebra Tiles



4. Solving linear equations in one variable

Take the equation 2x – 1 = 5 for an example. When modeling an equation using tiles, students can use a black separation bar to separate the two sides of the equation. Have the student place two violet rectangles and one small red square to the left of the bar, and five small lime squares to the right of the bar (Picture 4). Add one small lime square to each side of the bar. Because the newly added small lime square combined with the small red square forms a zero pair, both tiles can be removed. Thus, the equation becomes 2x = 6. Two violet rectangles should now be on the left side of the bar and two groups of three small lime squares should be on the right side of the bar. Divide each side of the equation by 2 (i.e., 2x/2 = 6/2), and remove one group of tiles from each side. The answer to this equation is x = 3.

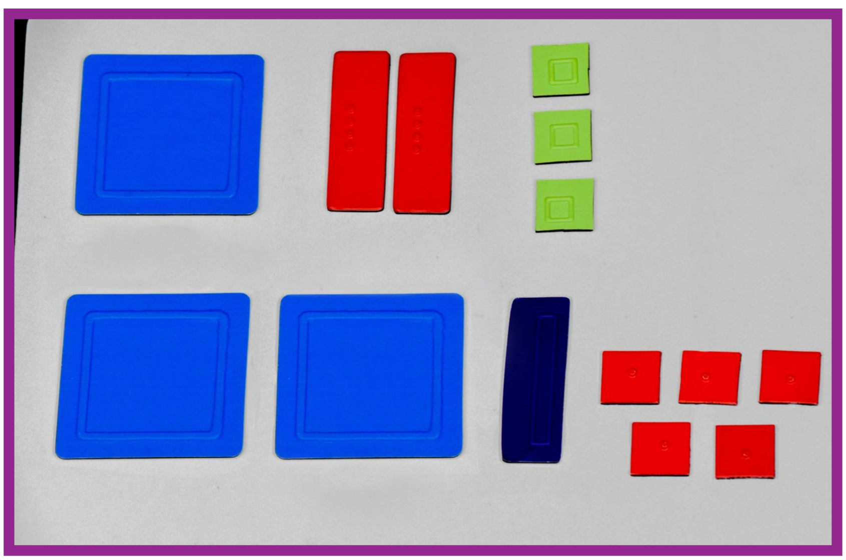


Picture 4: Modeling equation 2x - 1 = 5 using Tactile Algebra Tiles

5. Adding and subtracting polynomials

To add two polynomials, such as $x^2 - 2x + 3$ and $2x^2 + x - 5$, first model the polynomials using tiles. Have the student model one large blue square, two red rectangles, and three small lime squares to represent the first polynomial, and two large blue squares, one violet rectangle, and five small red squares to represent the second polynomial (Picture 5). To add these two polynomials, join the two groups of tiles and remove any zero pairs. Specifically, the student can remove one pair of red and violet rectangles and three pairs of small red and lime squares. Count the tiles remaining and get the sum of the two polynomials which is $3x^2 - x - 2$.

Picture 5: Modeling polynomials $x^2 - 2x + 3$ and $2x^2 + x - 5$ using Tactile Algebra Tiles



Take the following as an example of subtraction of polynomials: $x^2 - 2x - 3 - (2x^2 + 3x - 4)$. Because subtracting a polynomial can be done by adding its inverse, the problem becomes $x^2 - 2x - 3 + (-2x^2 - 3x + 4)$. Then follow the steps mentioned above to model the polynomials, join them, and remove any zero pairs. The answer to this problem is $-x^2 - 5x + 1$.

6. Multiplying polynomials

Take $(x + 2)(2x - 3)$ as an example. As with the multiplication of two integers problem, when multiplying two polynomials, begin by setting up a rectangle. The first factor, $x + 2$, is the length of the vertical side of the rectangle, and the second factor, $2x - 3$, is the length of the horizontal side. Then, use tiles to fill in the rectangle. Remind the student that the product of two values with the same sign is positive; if the signs are opposite, the product is negative. For instance, $x \times x = x^2$ (a large blue square), and $x \times (-1) = -x$ (a red rectangle). Specifically, from left to right and from top to bottom (as seen in Picture 6), the tiles filling up the rectangle are the following:

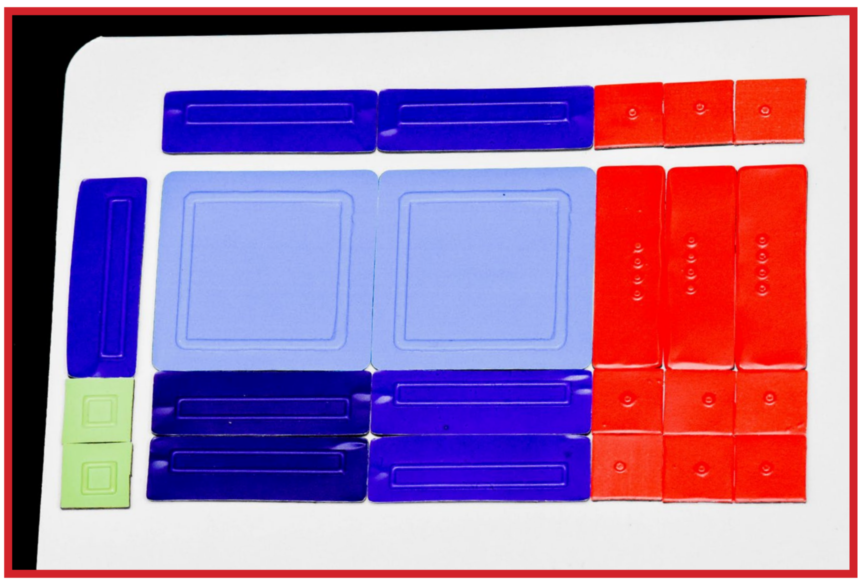
- top line: large blue square, large blue square, red rectangle, red rectangle, and red rectangle (rectangles are placed vertically);
- second line: violet rectangle, violet rectangle,

small red square, small red square, and small red square (rectangles are placed horizontally);

- bottom line: violet rectangle, violet rectangle, small red square, small red square, and small red square (rectangles are placed horizontally).

To find the answer to this multiplication problem, the student needs to remove any zero pairs in the rectangle and count all the tiles remaining. Three pairs of violet and red rectangles should be removed. The answer is $2x^2 + x - 6$.

Picture 6: Illustration of polynomial multiplication $(x + 2)(2x - 3)$ using Tactile Algebra Tiles



7. Factoring polynomials

Factoring a polynomial with algebra tiles involves setting up a rectangle properly and reading its dimensions. Take the polynomial $x^2 + 5x + 6$ as an example. First, model the polynomial using one large blue square, five violet rectangles, and six small lime squares. Next, arrange these tiles into a rectangle. Always begin with the first and last terms of a polynomial. Place the large blue tile at the upper left corner of the rectangle. Arrange the six small lime squares into a two-by-three array and place them at the lower right corner. Then, fill in the rest of the rectangle using the violet rectangles (Picture 7). When completed, arrangement of the tiles in the rectangle is the following:

- top line: large blue square, violet rectangle, violet rectangle, and violet rectangle (rectangles are placed vertically);
- second line: violet rectangle, small lime square, small lime square, and small lime square (rectangle is placed horizontally);
- bottom line: violet rectangle, small lime square, small lime square, and small lime square (rectangle is placed horizontally).

Picture 7: Factoring polynomial $x^2 + 5x + 6$ using Tactile Algebra Tiles

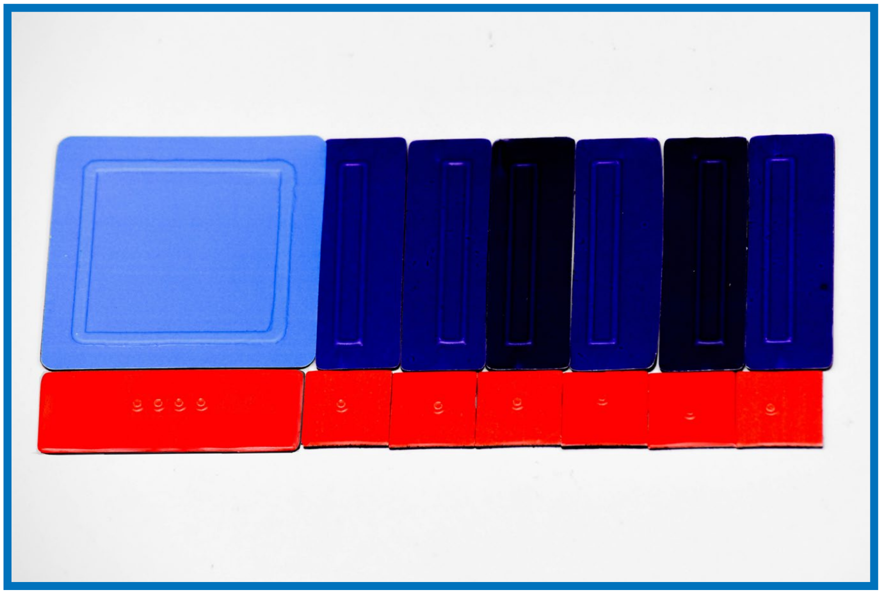
You may also want the student to try arranging the six small lime squares into a six-by-one array. The student will find that with this arrangement it is impossible to use five violet rectangles to complete the rectangle.

Last, read the dimensions of the rectangle created and find that its vertical side is $x + 2$ and its horizontal side is $x + 3$. Therefore, the answer to this factoring problem is $(x + 2)(x + 3)$.

Sometimes it is necessary to add zero pairs in order to arrange given tiles into a rectangle, as in factoring $x^2 + 5x - 6$. First, model the polynomial using one large blue square, five violet rectangles, and six small red squares. Next, place the large blue square at the upper left corner of the rectangle and the six small red squares at the lower right corner of the rectangle in a one-by-six array. Then, fill in using the rectangles. Because the five violet rectangles are not enough, a zero pair of violet and red rectangles must be added (Picture 8). When completed, the arrangement of the tiles in the rectangle is the following:

- top line: one large blue square and six violet rectangles (rectangles are placed vertically);
- bottom line: one red rectangle and six small red squares (rectangle is placed horizontally).

Read the dimensions of the rectangle created and find out that its vertical side is x – 1 and its horizontal side is x + 6. Therefore, the answer to this factoring problem is (x – 1)(x + 6).



Picture 8: Factoring polynomial $x^2 + 5x - 6$ using Tactile Algebra Tiles

Extension

Tactile Algebra Tiles can be used in combination with other tools available from APH to add more flexibility to the teaching and learning process. These tools include but are not limited to Expanded Beginner's Abacus (Catalog #: 1-03181-00), EZeeCOUNT Abacus (Catalog #: 1-03185-00), Orion TI-30XS MultiView Talking Scientific Calculator (Catalog #: 1-07330-00), and DRAFTSMAN Tactile Drawing Board (Catalog #: 1-08857-00). Teachers and parents can choose a combination of tools based on curricula and the students' needs.

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